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Find the partial fraction decomposition of $f(x) = \dfrac{3x+1}{(x-1)\cdot\left(x^2+1\right)}$

1) Because the denominator is a product of a linear factor, which is x-1, and a an irreducible quadratic, you can write the setup below.

$$\frac{3x+1}{(x-1)\left(x^2+1\right)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$$

2) Now multiply both sides by $(x-1)\left(x^2+1\right)$ to clear the denominators

$$(x-1)\cdot\left(x^2+1\right)\cdot\frac{3x+1}{(x-1)\cdot\left(x^2+1\right)} = \left[\frac{A}{(x-1)} + \frac{Bx+C}{x^2+1}\right](x-1)\cdot\left(x^2+1\right)$$

$$3x+1 = A\left(x^2+1\right) + (Bx+C)(x-1)$$

2a) Set x=1 to isolate A.

$$3(1)+1=A\left(1^2+1\right)+(B(1)+C)(1-1)$$

$$3+1=A(1+1)+(B+C)(0)$$

$$4=A(2)$$

$$2=A$$

2b) Set A=2 and x=0 to isolate C

$$3(0)+1=2\left(0^2+1\right)+(B(0)+C)(0-1)$$

$$1=2(1)+C(-1)$$

$$1=2-C$$

$$1-2=-C$$

$$1=C$$

2c) Set A=2, C=1 and x=2(this is arbitrary, meaning you can choose whatever value you want for x) to get B

$$3(2)+1=2\left(2^2+1\right)+(B(2)+1)(2-1)$$

$$6+1 = 2(4+1)+(2B+1)(1)$$

$$7=2(5)+2B+1$$

$$7=10+2B+1$$

$$7=11+2B$$

$$7-11=2B$$

$$-4=2B$$

$$-2=B$$

3) Now you have the constants, so you can write the final form as

$$\frac{3x+1}{(x-1)\cdot\left(x^2+1\right)} = \frac{2}{x-1} + \frac{-2x+1}{x^2+1}$$