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Find the parametric equations of the tangent line to the curve of intersection of the two surfaces shown below at the point (2,0,3)

S_1: $x^2 + y^2 = 4$ This a cylinder centered on the z axis

S_2: $x + z = 5$ This is a plane

1) Rewrite $x^2 + y^2 = 4$ as function of three variables: $f(x,y,z) = x^2 + y^2 - 4 = 0$
2) Rewrite $x+z=5$ as a function of three variables: $g(x,y,z) = x+z-5 = 0$
3) Form the gradient of f by taking partials and putting them inside a vector. Evaluate.

$$\nabla f(x,y,z) = <2x, 2y, 0>$$
$$\nabla f(2,0,3) = <2(2), 2(0), 0> = <4,0,0>$$

4) Form the gradient of g by finding its partials and putting them inside a vector. Evaluate.

$$\nabla g(x,y,z) = <1,0,1>$$
$$\nabla g(2,0,3) = <1,0,1>$$

5) Form the cross product of the gradient vectors. This cross product gives the directions we need to form the parametric equations of the tangent line.

$$\nabla f(2,0,3) \text{ x } \nabla g(2,0,3) = <4,0,0> \text{ x } <1,0,1> = \begin{pmatrix} i & j & k \\ 4 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix} = (0\cdot 1 - 0\cdot 0)i - (4\cdot 1 - 1\cdot 0)j + (4\cdot 0 - 1\cdot 0)k$$

$$= 0i - 4j + 0k = -4j$$

6) Parametric equations have the following standard form:

$x(t) = x_0 + at$ $y(t) = y_0 + bt$ $z(t) = z_0 + ct$

We have to identify a, b and c. These numbers are the components of the cross product shown above.

$a=0$ $b=-4$ $c=0$

$x_0 = 2$ $y_0 = 0$ $z_0 = 3$

$x(t) = 2+0t$ $y(t) = 0-4t$ $z(t) = 3+0t$

$x(t) = 2$ $y(t) = -4t$ $z(t) = 3$

[redacted]