Find the antiderivative of $\int x\ln(x+1)dx$

Key idea: Apply the concept of ILATE. This is how you choose u. In this case, this means u is the logarithmic function ln(x+1)

$$\int x\ln(x+1)dx$$

$u=\ln(x+1)$ $\qquad$ $dv=x$

$du=\dfrac{1}{x+1}dx$ $\qquad$ $v=\dfrac{1}{2}x^2$

2) Now setup the integral.

$$\int x\cdot\ln(x+1)dx=uv-\int vdu=\ln(x+1)\cdot\frac{1}{2}x^2-\int\frac{1}{2}x^2\cdot\frac{1}{1+x}dx$$

3) Now we can assemble the pieces together.

$$=\frac{x^2\ln(x+1)}{2}-\left(\int\frac{1}{2}x\ dx-\int\frac{1}{2}dx+\int\frac{1}{x+1}dx\right)$$

4) Now perform the integrations from 3) above.

$$=\frac{x^2\ln(x+1)}{2}-\left(\frac{1}{4}x^2-\frac{1}{2}x+\frac{1}{2}\ln|x+1\right)+C$$

5) Now distribute the negative -1.

$$=\frac{x^2\ln(x+1)}{2}-\frac{1}{4}x^2+\frac{1}{2}x-\frac{1}{2}\ln|x+1|+C$$

1) Rewrite this as shown: $\dfrac{1}{2}\cdot\dfrac{x^2}{1+x}$

2) Because the degree in the top is bigger, you have to divide.

$$\begin{array}{r|l} & x-1 \\ \hline x+1 & x^2+0x+0 \\ & -x^2-x \\ \hline & -x \\ & x+1 \\ \hline & 1 \end{array}$$

3) Now you can write $\dfrac{1}{2}\dfrac{x^2}{x+1}$ as

$$\frac{1}{2}\left(x-1+\frac{1}{x+1}\right)$$

$$\frac{1}{2}x-\frac{1}{2}+\frac{1}{2(x+1)}$$

If you practice every day, you'll learn all you need to learn. This is 100% guaranteed.