

$$\int e^{-x^4} dx$$

1) The series for e^x is $1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \dots$

2) Replace x with $-x^4$ to get the new series: $e^{-x^4} = 1 - x^4 + \frac{(-x^4)^2}{2} + \frac{(-x^4)^3}{6} + \dots$

3) Simplify this:

$$= 1 - x^4 + \frac{x^8}{2} - \frac{x^{12}}{6} + \dots$$

4) Now integrate this term by term to get

$$= x - \frac{x^5}{5} + \frac{x^9}{18} - \frac{x^{13}}{78} + \frac{x^{17}}{408} - \frac{x^{21}}{2520} + \frac{x^{25}}{18000} - \dots$$

5) The series in 4) above represents the antiderivative of e^{-x^4}