

You're told $3 \sin(x) + 3 \cos(y) - 2 \sin(x) \cos(y) + x = 4\pi$

You have to find the slope at $\left(4\pi, \frac{3\pi}{2}\right)$

1) Use implicit differentiation. Be sure to differentiate terms with y using the chain rule.

$$\frac{d}{dx}(3 \sin(x) + 3 \cos(y) - 2 \sin(x) \cos(y) + x) = \frac{d}{dx}(4\pi) \quad \text{Setup the derivative}$$

2) Now differentiate each term on both sides with respect to x . Use the chain rule on $\cos(y)$ and the product and chain rules on $-2\sin(x)\cos(y)$

$$3 \cos(x) - 3 \sin(y)y' - 2(\cos(x)\cos(y) + \sin(x)(-\sin(y)y')) + 1 = 0$$

3) Distribute the -2 into the parenthesis to prepare for solving for y' .

$$3 \cos(x) - 3 \sin(y)y' - 2 \cos(x)\cos(y) + 2 \sin(x)\sin(y)y' + 1 = 0$$

4) Now move terms without y' to the right side.

$$-3 \sin(y)y' + 2 \sin(x)\sin(y)y' = -1 + 2 \cos(x)\cos(y) - 3 \cos(x)$$

5) Now factor y' from each term on the left.

$$y'(-3 \sin(y) + 2 \sin(x)\sin(y)) = -1 + 2 \cos(x)\cos(y) - 3 \cos(x)$$

6) Now divide both sides by the expression on y'

$$y' = \frac{-1 + 2 \cos(x) \cos(y) - 3 \cos(x)}{-3 \sin(y) + 2 \sin(x) \sin(y)}$$

7) Now evaluate this ugly mess at the point stated in the question:

$$y' \left(4\pi, \frac{3\pi}{2}\right) = \frac{-1 + 2 \cos(4\pi) \cdot \cos\left(\frac{3\pi}{2}\right) - 3 \cos(4\pi)}{-3 \sin\left(\frac{3\pi}{2}\right) + 2 \sin(4\pi) \cdot \sin\left(\frac{3\pi}{2}\right)} = \frac{-1 + 2(1) \cdot (0) - 3 \cdot (1)}{-3(-1) + 2 \cdot 0 \cdot -1} = \frac{-1 - 0 - 3}{3} = \frac{-4}{3}$$

8) Therefore the slope is $\frac{-4}{3}$.