

Find the derivative of $f(x) = x \cdot \sqrt{64 - x^2}$

$$1) f'(x) = \frac{d}{dx} \left(x \cdot \sqrt{64 - x^2} \right)$$

2) Now you differentiate using the product and chain rules.

The product rule states that $(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$.

The chain rule states that $\frac{d}{dx} f(g(x)) = f'(g(x)) g'(x)$

$$f'(x) = (1) \cdot \sqrt{64 - x^2} + x \cdot \frac{1}{2} (64 - x^2)^{\frac{-1}{2}} \cdot (-2x)$$

3) Now this has to be cleaned up.

$$f'(x) = \sqrt{64 - x^2} + \frac{x}{2} \cdot (-2x) \cdot \frac{1}{\sqrt{64 - x^2}}$$

$$f'(x) = \sqrt{64 - x^2} - \frac{x^2}{\sqrt{64 - x^2}}$$

$$f'(x) = \frac{\sqrt{64 - x^2} \cdot \sqrt{64 - x^2}}{\sqrt{64 - x^2}} - \frac{x^2}{\sqrt{64 - x^2}}$$

$$f'(x) = \frac{64 - x^2 - x^2}{\sqrt{64 - x^2}} = \frac{64 - 2x^2}{\sqrt{64 - x^2}}$$