Find the direction of maximum increase of $f(x,y)=x^3y^2$ at the point (1,-3).

1) Form the gradient. This means put the partial derivatives of f inside a vector.

$$\nabla f(x,y)=<\frac{\partial}{\partial x}x^3y^2, \frac{\partial}{\partial y}x^3y^2>$$

$$\nabla f(x,y)=<3x^2y^2, 2x^3y>$$

Each partial is found using the power rule while holding the other variable constant.

2) Evaluate the gradient at the point (1,-3).

$$\begin{aligned}\nabla f(1,-3)&=<3(1)^2(-3)^2, 2(1)^3(-3)>\\&=<3(1)(9), 2(1)(-3)>\\&=<27,-6>\end{aligned}$$

3) Find the magnitude of the gradient by using the Pythagorean Theorem.

$$||\nabla f(1,-3)||=\sqrt{27^2+(-6)^2}=\sqrt{765}=\sqrt{9\times 85}=3\sqrt{85}$$

4) Divide the gradient by its magnitude to find the direction of maximum increase.

$$\begin{aligned}\text{Direction of maximum increase at } (1,-3)&=\frac{\nabla f(1,-3)}{||\nabla f(1,-3)||}\\&=\frac{<27,-6>}{3\sqrt{85}}\\&=<\frac{27}{3\sqrt{85}},\frac{-6}{3\sqrt{85}}>\\&=<\frac{9}{\sqrt{85}},\frac{-2}{\sqrt{85}}>\\&=<\frac{9}{\sqrt{85}}\times\frac{\sqrt{85}}{\sqrt{85}},\frac{-2}{\sqrt{85}}\times\frac{\sqrt{85}}{\sqrt{85}}>\\&=<\frac{9\sqrt{85}}{85},\frac{-2\sqrt{85}}{85}>\end{aligned}$$