1) $\frac{dy}{dx} = y^2 - 1$

Separate the variables

2) $\frac{dy}{y^2 - 1} = dx$

3) $\frac{1}{(y-1)(y+1)} dy = dx$

Factor $y^2 - 1$

4) $\int \frac{1}{(y-1)\cdot(y+1)} \, dy = x + C$

Integrate the right side to get x+C. On the left, you have to do partial fractions.

5) $\frac{1}{(y-1)\cdot(y+1)} = \frac{A}{y-1} + \frac{B}{y+1}$

Setup the partial fractions

Multiply both sides by (y-1)(y+1) to clear the denominators

1=A(y+1)+B(y-1)

set y=-1 to get B	set y=1 to get A
1=A(-1+1)+B(-1-1)	1=A(1+1)+B(1-1)
1=A(0)-2B	1=A(2)+B(0)
1=-2B	1=2A
$B = \frac{-1}{2}$	$A = \frac{1}{2}$

6) $\int \frac{1}{2(y-1)} - \frac{1}{2(y+1)} \, dy = \frac{1}{2} \cdot \int \frac{1}{y-1} - \frac{1}{y+1} \, dy = \frac{1}{2}\ln|y-1| - \frac{1}{2}\ln|y+1|$

7) $\frac{1}{2} \ln|y-1| - \frac{1}{2}\ln|y+1| = x + C$

Apply a basic rule of logs $\ln(a) - \ln(b) = \ln\left(\frac{a}{b}\right)$

8) $\frac{1}{2}\ln\left|\frac{y-1}{y+1}\right| = x + C$