Find the maximum of $f(x)=\frac{-1}{2}x^2 + 4x + 13$

1) The maximum occurs where the first derivative is 0.

2) $f'(x)=\frac{d}{dx}\left(\frac{-1}{2}x^2 + 4x + 13\right)$ Setup the deriv ativ e

$=\frac{d}{dx}\left(\frac{-1}{2}x^2\right) + \frac{d}{dx}(4x) + \frac{d}{dx}(13)$ Distribute the derivative across the parenthesis

$=\frac{-1}{2}(2)x^{2-1} + 4$ Apply the power rule. Bring the 2 down, and

$= \frac{-2}{2}x+4$ subtract 1 from the exponent.

$=-x + 4$ Now you have the derivative.

3) Set the derivative equal to zero, and solve for the critical value.

$-x + 4 = 0$ Set $f'(x)=0$

$-x=-4$ Subtract 4 form both sides

$x=4$ Divide both sides by -1

4) Differentiate again to look at the second derivative.

$f''(x)=\frac{d}{dx}f'(x)=-1$

Because the second derivative is negative, $x=4$ is where the maximum value occurs.

5) To find the maximum value, evaluate the function at $x=4$

$f(4)=\frac{-1}{2}4^2 + 4(4) + 13$ Replace x with 4

$=\frac{-1}{2} \cdot 16 + 16 + 13$ Square 4

$=-8+16+13$ Divide 16 by 2 to get 8

$=21$ Add