Find the following limit:

$$\lim_{x \to 0} \frac{e^x - 1 - x}{x^2}$$

1) First we check what happens if we could plug 0 into the functions. In this case, we have an indeterminate form.

$$\frac{e^0 - 1 - 0}{0^2} = \frac{1-1}{0} = \frac{0}{0}$$

This is an indeterminate form. Remember that plugging 0 into the functions is not actually legal mathematically, but we do it in order to get a feel for what we should do.

2) Because we have an indeterminate form, we apply L'Hopital.

$$\lim_{x \to 0} \frac{e^x - 1 - x}{x^2} = \lim_{x \to 0} \frac{\frac{d}{dx}\left(e^x - 1 - x\right)}{\frac{d}{dx}(x^2)} = \lim_{x \to 0} \frac{e^x - 1}{2x}$$

3) We replace x with 0, and check again.

$$\frac{e^0 - 1}{2(0)} = \frac{1-1}{0} = \frac{0}{0}$$

This is an indeterminate form again. So apply L'Hopital again.

4) Differentiate top and bottom, as before, and check.

$$\lim_{x\to 0} \frac{\frac{d}{dx}\left(e^x - 1\right)}{\frac{d}{dx}(2x)} = \lim_{x\to 0} \frac{e^x}{2}$$

Now we can replace x with 0, and find that the limit is

$$\lim_{x\to 0} \frac{e^x}{2} = \frac{1}{2} \lim_{x\to 0} e^x = \frac{1}{2}\left(e^0\right) = \frac{1}{2}(1) = \frac{1}{2}$$