

Find the limit shown below.

$$\lim_{x \rightarrow 1} \frac{x-1}{\sqrt{x+3}-2}$$

1) If you replace  $x$  with one, you get  $\frac{1-1}{\sqrt{1+3}-2} = \frac{0}{\sqrt{4}-2} = \frac{0}{2-2} = \frac{0}{0}$

2) To remedy this, multiply top and bottom by the conjugate of  $\sqrt{x+3}-2$ . This conjugate is  $\sqrt{x+3}+2$

3)  $\lim_{x \rightarrow 1} \frac{(x-1)}{\sqrt{x+3}-2} = \lim_{x \rightarrow 1} \left[ \frac{(x-1)}{\sqrt{x+3}-2} \cdot \frac{\sqrt{x+3}+2}{\sqrt{x+3}+2} \right]$  setup the multiplication

$$\lim_{x \rightarrow 1} \frac{(x-1)}{\sqrt{x+3}-2} = \lim_{x \rightarrow 1} \frac{(x-1)(\sqrt{x+3}+2)}{(\sqrt{x+3})^2 + 2\sqrt{x+3} - 2\sqrt{x+3} - 4}$$
 multiply in the bottom

$$\lim_{x \rightarrow 1} \frac{(x-1)}{\sqrt{x+3}-2} = \lim_{x \rightarrow 1} \frac{(x-1)(\sqrt{x+3}+2)}{x+3-4}$$
 middle terms in bottom cancel

$$\lim_{x \rightarrow 1} \frac{(x-1)}{\sqrt{x+3}-2} = \lim_{x \rightarrow 1} \frac{(x-1)(\sqrt{x+3}+2)}{(x-1)}$$
 cancel  $x-1$  in the top with the one in the bottom

$$\lim_{x \rightarrow 1} \frac{(x-1)}{\sqrt{x+3}-2} = \lim_{x \rightarrow 1} (\sqrt{x+3}+2) = \sqrt{1+3}+2 = \sqrt{4}+2=2+2=4$$
 replace  $x$  with 1 to get 4