

Find the equation of the tangent line to $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 4$ at the point $(-3\sqrt{3}, 1)$.

1) Use implicit differentiation: $\frac{2}{3}x^{\frac{2}{3}-1} + \frac{2}{3}y^{\frac{2}{3}-1} \cdot y' = 0$ Bring the $\frac{2}{3}$ down and subtract 1. Be sure to multiply by y' from the chain rule.

2) Rewrite: $\frac{2}{3}x^{\frac{2}{3}-1} + \frac{2}{3}y^{\frac{2}{3}-1} \cdot y' = 0$ Setup the subtraction. $\frac{2}{3} - 1 = \frac{2}{3} - \frac{3}{3} = \frac{-1}{3}$

3) Complete the subtractions: $\frac{2}{3}x^{\frac{-1}{3}} + \frac{2}{3}y^{\frac{-1}{3}} \cdot y' = 0$

4) Now isolate y' by subtracting $\frac{2}{3}x^{\frac{-1}{3}}$: $\frac{2}{3}y^{\frac{-1}{3}} \cdot y' = \frac{-2}{3}x^{\frac{-1}{3}}$

5) Multiply both sides by $\frac{3}{2}$: $y^{\frac{-1}{3}} \cdot y' = -x^{\frac{-1}{3}}$ This clears the fractions

6) Now multiply both sides by $y^{\frac{1}{3}}$: $y' = -x^{\frac{-1}{3}} \cdot y^{\frac{1}{3}}$ This puts $y^{\frac{1}{3}}$ on the right and gets rid of it of from the left. $y^{\frac{1}{3}} \cdot y^{\frac{-1}{3}} = y^{\frac{1}{3}-\frac{1}{3}} = y^0 = 1$

7) You can rewrite this as shown This is y' : $y' = \frac{-\sqrt[3]{y}}{\sqrt[3]{x}}$

8) Evaluate y' at the point $(-3\sqrt{3}, 1)$: $y' = \frac{\sqrt[3]{1}}{\sqrt[3]{-3\sqrt{3}}} = \frac{-1}{-\sqrt{3}} = \frac{1}{\sqrt{3}}$

9) Now setup and simplify the equation of the tangent line: $y - 1 = \frac{1}{\sqrt{3}}(x - (-3\sqrt{3}))$

$$y = \frac{1}{\sqrt{3}}x + \frac{3\sqrt{3}}{\sqrt{3}} + 1$$

$$y = \frac{1}{\sqrt{3}}x + 3 + 1$$

$$y = \frac{1}{\sqrt{3}}x + 4$$