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$f(x)=\ln\left(x\cdot\sqrt{x^2-1}\right)$

1) $f(x)=\ln(x)+\ln\left(\sqrt{x^2-1}\right)$ rewrite using the fact that ln(f(x)g(x))=ln(f(x))+ln(g(x))

2) $f'(x)=\frac{d}{dx}\ln(x)+\frac{d}{dx}\ln\left(\sqrt{x^2-1}\right)$ Apply the derivative symbol to each term individually

3) $\frac{d}{dx}\ln(x)=\frac{1}{x}$ this is the easy one

4) $\frac{d}{dx}\ln\left(\sqrt{x^2-1}\right)=\frac{d}{dx}\ln\left[\left(x^2-1\right)^{\frac{1}{2}}\right]=\frac{1}{2}\cdot\frac{d}{dx}\cdot\ln\left(x^2-1\right)$ Put $\frac{1}{2}$ in front using a rule of logs $\ln\left(x^a\right)=a\ln(x)$

5) Now use the chain rule: $\frac{1}{2}\cdot\frac{d}{dx}\ln\left(x^2-1\right)=\frac{1}{2}\cdot\frac{1}{x^2-1}\cdot\left[\frac{d}{dx}\left(x^2-1\right)\right]=\frac{1}{2}\cdot\frac{1}{x^2-1}\cdot 2x=\frac{2}{2}\cdot\frac{x}{x^2-1}=\frac{x}{x^2-1}$

6) Now add the results of steps 3) and 5) together to get the final derivative: $f'(x)=\frac{1}{x}+\frac{x}{x^2-1}$

7) Rewrite the final expression on the right by getting a common denominator:

7a) This means $\frac{1}{x}$ has to be multiplied by $\frac{x^2-1}{x^2-1}$ to get $\frac{x^2-1}{x\left(x^2-1\right)}$

7b) This means $\frac{x}{x^2-1}$ has to be multiplied by $\frac{x}{x}$ to get $\frac{x\cdot x}{x\left(x^2-1\right)}=\frac{x^2}{x\left(x^2-1\right)}$

8) Now combine these two to get $f'(x)=\frac{x^2-1}{x\left(x^2-1\right)}+\frac{x^2}{x\left(x^2-1\right)}=\frac{x^2-1+x^2}{x\left(x^2-1\right)}=\frac{2x^2-1}{x\left(x^2-1\right)}$

Note: Fight on!! You don't know what you can do until you give everything you've got, which is way more than you think!!