

1) The period is given by: $T=2\pi\sqrt{\frac{L}{g}}$

2) Divide by 2π on both sides:

$$\frac{T}{2\pi} = \frac{2\pi}{2\pi} \sqrt{\frac{L}{g}} \quad \Rightarrow \quad \frac{T}{2\pi} = \sqrt{\frac{L}{g}}$$

3) Square both sides:

$$\left(\frac{T}{2\pi}\right)^2 = \frac{L}{g} \quad \Rightarrow \quad \frac{T^2}{4\pi^2} = \frac{L}{g}$$

4) Flip the equation to isolate g and multiply both sides by L

$$\frac{4\pi^2}{T^2} = \frac{g}{L} \quad \Rightarrow \quad g = \frac{4\pi^2}{T^2} L$$

5) g is also given by $g = \frac{GM}{r^2}$, so you can set this g equal to the one from above

$$\frac{GM}{r^2} = \frac{4\pi^2}{T^2} L$$

6) Now multiply both sides by r^2 and then divide by big G

$$M = \frac{4\pi^2}{T^2} L \left(\frac{r^2}{G}\right)$$

7) Now you can plug in the information given.

8) If $r=7550$ km, $T=10$ seconds, $L=0.5$ meters, you get $M = \frac{4\pi^2}{10^2} (.5) \left(\frac{(7550 \times 1000)^2}{6.67 \times 10^{-11}}\right) = 1.687 \times 10^{23}$ kg