

Partial fraction decomposition of $\frac{1}{x^2 - 5x + 6}$

1) First factor the bottom, as shown.

$$\frac{1}{(x-3) \cdot (x-2)}$$

2) Now we see the denominator is a product of two linear factors.

2a) Write the setup for partial fractions.

$$\frac{1}{(x-3) \cdot (x-2)} = \frac{A}{x-3} + \frac{B}{x-2}$$

2b) Multiply both sides by $(x-3)(x-2)$. This will clear the denominator on the left.

$$(x-3)(x-2) \cdot \frac{1}{(x-3) \cdot (x-2)} = \left[\frac{A}{x-3} + \frac{B}{x-2} \right] \cdot (x-3)(x-2)$$

$$\frac{(x-3)(x-2)}{(x-3) \cdot (x-2)} \cdot 1 = \frac{A}{x-3} \cdot (x-3) \cdot (x-2) + \frac{B}{x-2} \cdot (x-3) \cdot (x-2)$$

$$1 = A(x-2) + B(x-3)$$

3) Now you can solve for each of A and B.

Set $x=2$. This will give you B.

Set $x=3$. This will give you A.

$$1 = A(2-2) + B(2-3)$$

$$1 = A(3-2) + B(3-3)$$

$$1 = A(0) - B$$

$$1 = A(1) + B(0)$$

$$1 = -B$$

$$1 = A$$

$$-1 = B$$

4) Now you can write the partial fraction decomposition as shown.

$$\frac{1}{x^2 - 5x + 6} = \frac{1}{x-3} - \frac{1}{x-2}$$