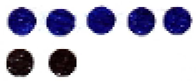
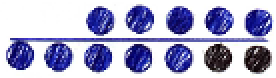
1) The ratio of blue marbles to black marbles is 5 to 2. What's the chance of picking a blue marble?

The picture below shows there are seven marbles in all. This means the probability of picking a blue one is



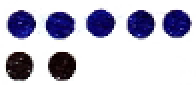
$$\frac{\text{favorable}}{\text{total}} = \frac{5}{7}$$

$$P(\text{blue}) = \frac{\text{blue}}{\text{blue} + \text{black}}$$



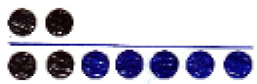
2) The ratio of blue marbles to black marbles is 5 to 2. What's the chance of picking a black marble?

The picture below shows there are seven marbles in all. This means the probability of picking a black one is



$$\frac{\text{favorable}}{\text{total}} = \frac{2}{7}$$

$$P(\text{black}) = \frac{\text{black}}{\text{blue} + \text{black}}$$



Note that in the two preceding examples, parts to parts were compared, and this means that the total number of items is the sum of the parts. This number becomes the denominator when finding a probability.

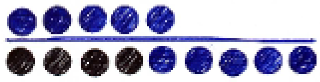
$$P(\text{part}_1) = \frac{\text{part}_1}{\text{part}_1 + \text{part}_2}$$

3) If the number of black marbles is doubled, what is the new ratio of blue to black?

To double means to multiply by two, so now the marbles are as laid out below. This means there are four blacks.

4) Given the updated number of red marbles, what's the probability of picking a blue one this time. Compare this probability with the probability found in question 1) above.

$$P(\text{blue}) = \frac{5}{9} = \frac{\text{blue}}{\text{blue} + \text{black}}$$



Be sure to change the bottom here from 7 to 9 because the total number of numbers is different now.

To compare the two probabilties, we can just form a simple ratio, as shown below.

$$\frac{P(\text{blue}_{\text{new}})}{P(\text{blue}_{\text{old}})} = \frac{\frac{5}{9}}{\frac{5}{7}} = \frac{5}{9} \cdot \frac{7}{5} = \frac{7}{9}$$

$$P(\text{blue}_{\text{new}}) = \frac{7}{9} \cdot P(\text{blue}_{\text{old}})$$

5) Given the updated number of red marbles, what's the probability of picking a black one this time. Compare this probability with the probability found in question 1) above.

$$P(\text{black}) = \frac{4}{9} = \frac{\text{black}}{\text{blue} + \text{black}}$$

Be sure to change the bottom here from 7 to 9 because the total number of numbers is different now. To compare the two probabilties, we can just form a simple ratio, as shown below.

$$\frac{P(\text{red}_{\text{new}})}{P(\text{red}_{\text{old}})} = \frac{\frac{4}{9}}{\frac{2}{7}} = \frac{4}{9} \cdot \frac{7}{2} = \frac{2 \cdot 2}{9} \cdot \frac{7}{2} = \frac{14}{9} = 1.56$$

$$P(\text{red}_{\text{new}}) = \frac{14}{9} \cdot P(\text{red}_{\text{old}}) = 1.56 \cdot P(\text{red}_{\text{old}})$$

The new probability is NOT double the old probability.

6) If the number of red marbles is scaled by a factor x, what's the impact on the probabilities of choosing a black or choosing a blue? Here r is the number of blacks, and b is the number of blues.

To scale means to multiply, so the new number of blacks is xr.

$P(\text{black}) = \frac{\text{blacks}}{\text{total}} = \frac{xr}{xr+b}$ $\qquad$ $P(\text{blue}) = \frac{\text{blues}}{\text{total}} = \frac{b}{xr+b}$

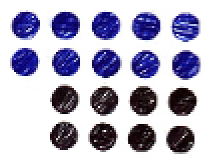
Be sure to change the denominator to reflect the new number total.

---

7) If the number of blue and the number of red are both scaled by a factor x, is there any impact on the probabilities?

Pretend at first to double the reds and blues.

Say this is the original. $\qquad$ Say this is the final.



$P(\text{black}) = \frac{4}{9}$

$P(\text{blue}) = \frac{5}{9}$

$\frac{\text{black}}{\text{blue}} = \frac{4}{5}$ $\qquad$ $P(\text{black}) = \frac{8}{18} = \frac{2\cdot 4}{2\cdot 9} = \frac{4}{9}$

$\frac{\text{blue}}{\text{black}} = \frac{5}{4}$ $\qquad$ $P(\text{blue}) = \frac{10}{18} = \frac{2\cdot 5}{2\cdot 9} = \frac{5}{9}$

In General: $\qquad$ $\frac{\text{black}}{\text{blue}} = \frac{8}{10} = \frac{2\cdot 4}{2\cdot 5} = \frac{4}{5}$

$P(\text{black}) = \frac{xr}{xr+xb} = \frac{x(r)}{x(r+b)} = \frac{r}{r+b}$

$\frac{\text{blue}}{\text{black}} = \frac{10}{8} = \frac{2\cdot 5}{2\cdot 4} = \frac{5}{4}$

$P(\text{blue}) = \frac{xb}{xr+xb} = \frac{x(b)}{x(r+b)} = \frac{b}{r+b}$

It's important to understand that physically, there is now indeed a new number of marbles. However, mathematically, the ratios and probabilities remain unchaged, as shown above.

---

8) If the current probability of picking a blue is as shown in the picture below, how many reds must be added to change that probability to half of what it is now?

$P(\text{blue}) = \frac{3}{5} = \text{current}$

Add 1 black

$P(\text{blue}) = \frac{3}{6}$

Add 2 black

$P(\text{blue}) = \frac{3}{7}$

Add 3 black

$P(\text{blue}) = \frac{3}{8}$

Add 4 black

$P(\text{blue}) = \frac{3}{9}$

Add 5 black

$P(\text{blue}) = \frac{3}{10} = \frac{1 \cdot 3}{2 \cdot 5} = \frac{1}{2} \cdot \frac{3}{5} = \text{half} \cdot \text{of} \cdot \frac{3}{5}$

Solve with Equation

$$\frac{\text{blue}}{\text{total} + \text{new} \cdot \text{black}} = \frac{1}{2} \cdot \left(\frac{3}{5}\right)$$

$$\frac{3}{5+x} = \frac{1}{2} \cdot \frac{3}{5}$$

$$\frac{3}{5+x} = \frac{3}{10}$$

$$3 \cdot 10 = 3(5+x)$$

$$10 = 5 + x$$

$$10 - 5 = x$$

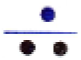
$$x = 5$$

Adding one new black marble at a time, and observing the probabilities, shows we must add 5 blacks.

9)The current ratio of blue to all marbles in a container is as shown below. Marbles are added in the ratio of 1 blue to 2 black per minute. How many marbles are blue after 3 minutes?

t=0

Now, since marbles are added in the ratio of 1 blue to 2 black per minute, this means that every minute 3 new marbles are added. This is shown below.



3 marbles are added per minute

Now add the 1 new blue to the 3 old blue, and the 2 new black to the old 2 black, to get the counts show below.

t=1

Now another minute passes, and three new marbles are added. This means now the counts look as shown below. Remember all you're doing is adding 1 blue and 2 black.

t=2

Now another minute passes, and three new marbles are added. This means now the counts look as shown below. Remember all you're doing is adding 1 blue and 2 black.

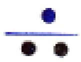
t=3

As the picture above shows, there are now 6 blues.

10) The current ratio of blue to all marbles in a container is as shown below. Marbles are added in the ratio of 1 blue to 2 black per minute. How many marbles are black after 3 minutes?

t = 0

Now, since marbles are added in the ratio of 1 blue to 2 black per minute, this means that every minute 3 new marbles are added. This is shown below.



3 marbles are added per minute

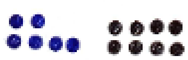
Now add the 1 new blue to the 3 old blue, and the 2 new black to the old 2 black, to get the counts show below.

t = 1

Now another minute passes, and three new marbles are added. This means now the counts look as shown below. Remember all you're doing is adding 1 blue and 2 black.

t = 2

Now another minute passes, and three new marbles are added. This means now the counts look as shown below. Remember all you're doing is adding 1 blue and 2 black.



t = 3

As the picture above shows, there are now 8 black.

11) The current ratio of blue to all marbles in a container is as shown below. Marbles are added in the ratio of 1 blue to 2 black per minute. Write a function to represent the number of blue marbles at any time t.

At t=0, the blues number 3. Think of this as the y intercept.

Then because blues increase at the rate of 1 per minute, you can write a linear function to represent the count of blue marbles as follows:

$$b(t) = 3 \cdot blue + \frac{1 \cdot blue}{1 \cdot minute} \cdot t$$

Without units this is $b(t) = 3 + 1 \cdot t$

---

12) Evaluate the function developed in question 11) above at t=0, t=1, t=2 and t=3, and comment on the meaning of the values.

$f(0) = 3 + 1(0) = 3$ This tells us at the start there are 3 blues.

$f(1) = 3 + 1(1) = 3 + 1 = 4$

This tells us that one second after the start, there are 4 blues.

$f(2) = 3 + 1(2) = 3 + 2 = 5$

This tells us that two seconds after the start, there are 5 blue marbles.

$f(3) = 3 + 1(3) = 3 + 3 = 6$

This tells us that three seconds after the start, there are 6 blue marbles.

These values are first found using pictures above.

---

13) The current ratio of blue to all marbles in a container is as shown below. Marbles are added in the ratio of 1 blue to 2 black per minute. Write a function to represent the number of black marbles at any time t.

At t=0, the reds number 2. Think of this as the y intercept.

Then because black increase at the rate of 2 per minute, you can write a linear function to represent the count of black marbles as follows:

$r(t) = 2 \cdot black + \frac{2 \cdot black}{1 \cdot minute} \cdot t$

Without units this is $r(t) = 2 + 2 \cdot t$

---

14) Evaluate the function developed in question 13) above at t=0, t=1, t=2 and t=3, and comment on the meaning of the values.

$f(0) = 2 + 1(0) = 2$ This tells us at the start there are 2 reds.

$f(1) = 2 + 2(1) = 2 + 2 = 4$

This tells us that one second after the start, there are 4 blacks.

$f(2) = 2 + 2(2) = 2 + 4 = 6$

This tells us that two seconds after the start, there are 6 black marbles.

$f(3) = 2 + 2(3) = 2 + 6 = 8$

This tells us that three seconds after the start, there are 8 black marbles.

These values are first found using pictures above.

---

15)The current ratio of blue to all marbles in a container is as shown below. Marbles are added in the ratio of 1 blue to 2 red per minute.Write a function to represent the total number of marbles in the container.

According to the development above, we know that any time t, there are b(t)=3+1t blue marbles, and r(t)=2+2t black marbles. Therefore the total number of marbles is just the sum of these, as shown below.

$$\text{total}(t)=b(t)+r(t)=3+1\cdot t+2+2\cdot t=5+3\cdot t$$

---

16)The current ratio of blue to all marbles in a container is as shown below. Marbles are added in the ratio of 1 blue to 2 black per minute.Write a function to represent the probability of choosing a blue marble at any time t.

Following the logic developed in the examples above, we can write

$$\frac{\text{blue}}{\text{all}}=\frac{3+1\cdot t}{(3+1\cdot t)+(2+2\cdot t)}=\frac{3+t}{5+3\cdot t}=P_b(t)$$

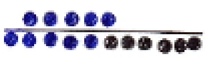
The denominator is the sum of the counts of the blues and blacks at any time t. This simply tells the total number of marbles at any time t.

17) Evaluate the function developed in question 16) above at t=0, t=1, t=2 and t=3, and comment on the meaning of the values.

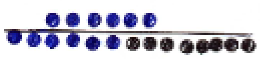
$P_b(0)=\frac{3+0}{5+0}=\frac{3}{5}$ This represents the probability of choosing a blue at the start.

$P_b(1)=\frac{3+1(1)}{5+3(1)}=\frac{4}{8}$ This represents the probability of choosing a blue 1 minute into the processing of adding marbles.

$P_b(2)=\frac{3+1(2)}{5+3(2)}=\frac{5}{11}$ This represents the probability of choosing a blue 2 minutes into the processing of adding marbles.



$P_b(3)=\frac{3+1(3)}{5+3(3)}=\frac{6}{14}$ This represents the probability of choosing a blue 3 minutes into the processing of adding marbles.



18) The current ratio of black to all marbles in a container is as shown below. Marbles are added in the ratio of 1 blue to 2 black per minute. Write a function to represent the probability of choosing a black marble at any time t.

Following the logic developed in the examples above, we can write

$$\frac{black}{all} = \frac{2+2\cdot t}{(3+1\cdot t)+(2+2\cdot t)} = \frac{2+2t}{5+3\cdot t} = P_r(t)$$

The denominator is the sum of the counts of the blues and blacks at any time t. This simply tells the total number of marbles at any time t.

---

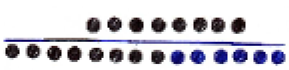
19) Evaluate the function developed in question 16) above at t=0, t=1, t=2 and t=3, and comment on the meaning of the values.

$P_r(0) = \frac{2+0}{5+0} = \frac{2}{5}$ This represents the probability of choosing a black at the start.

$P_r(1) = \frac{2+2(1)}{5+3(1)} = \frac{4}{8}$ This represents the probability of choosing a black 1 minute into the processing of adding marbles.

$P_r(2) = \frac{2+2(2)}{5+3(2)} = \frac{6}{11}$ This represents the probability of choosing a black 2 minutes into the processing of adding marbles.

$P_r(3) = \frac{2+2(3)}{5+3(3)} = \frac{8}{14}$ This represents the probability of choosing a black 3 minutes into the processing of adding marbles.



20) Now a question for you. Can you find the limiting probability of choosing a black marble? This is the probability you expect once a huge number of new blacks and blues have been added. You can solve this visually, or algebraically by writing a sequence of probabilities, and observing where it's tending. You can also plot the function

$P_r(t)$