

Find a unit vector in the direction of $u = \langle -2, 4 \rangle$

1) First find the magnitude of u using the Pythagorean Theorem.

$$\begin{aligned}
 \text{magnitude of } u &= \sqrt{(-2)^2 + (4)^2} && \text{Pythagorean Theorem} \\
 &= \sqrt{4 + 16} && \text{Square} \\
 &= \sqrt{20} && \text{Add} \\
 &= \sqrt{4 \cdot 5} && \text{Rewrite 20 as 4 times 5} \\
 &= \sqrt{4} \cdot \sqrt{5} && \text{Apply a basic rule of roots} \\
 &= 2\sqrt{5} && \text{The square root of 4 is 2}
 \end{aligned}$$

2) Now divide the vector u by its magnitude to find the unit vector along u .

$$\begin{aligned}
 \text{unit vector} &= \frac{\langle -2, 4 \rangle}{2\sqrt{5}} && \text{Setup the division by the magnitude} \\
 &= \left\langle \frac{-2}{2\sqrt{5}}, \frac{4}{2\sqrt{5}} \right\rangle && \text{Distribute to each component} \\
 &= \left\langle \frac{-1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \right\rangle && \text{Simplify by dividing -2 by 2, and 4 by 2} \\
 &= \left\langle \frac{-1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}, \frac{2}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} \right\rangle && \text{Setup the rationalization} \\
 &= \left\langle \frac{-\sqrt{5}}{5}, \frac{2\sqrt{5}}{5} \right\rangle && \text{Complete the rationalization}
 \end{aligned}$$